

Strength Prediction Model for Fly Ash Concrete

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Abstract - In this era of sustainable development the environmental ability to meet the present and future needs is a growing concern throughout the world. The present investigation involves production of medium to high strength concrete and studying the effect of incorporating fly ash on the strength development. Concrete mixes are proportioned to achieve the desired strength at a specified age. Hence some relationship allowing strength prediction at a specified age will be highly useful. Such a strength predicting relationship is available for control concrete (Abram's law) but not for concrete with mineral admixtures. The present work is a limited humble effort in this direction. Identifying the significant parameters affecting strength, statistical analysis has been performed to study the effect of the same. Using 'All Possible Regression' method of statistical analysis the present database has been analyzed and their significance examined by using simple statistical tools like 'R', 's' statistics, 'F' and 't' tests. These models being simple and user friendly might serve as useful tools for strength predictions.

Index Terms— Admixture, Compressive strength, Fly ash, High strength concrete, Regression analysis, Strength modeling, Water-cementitious material ratio

1 INTRODUCTION

Since wide scale coal firing for power generation began in 1920s, many millions tons of waste products have been generated causing a great threat to the environment. Fly ash is one such waste whose abundant availability throughout the world has rekindled the interest of incorporating it as an essential cementitious component in concrete. Fly ash particles are considered to be highly contaminating, due to their enrichment in potentially toxic trace elements which condense from the flue gas. Research on the potential applications of these wastes has environmental relevance, in addition to industrial interest. Most of the fly ash which is produced is disposed off as landfill, a practice which is under examination for environmental concerns. Disposal of fly ash will soon be too costly - if not forbidden. Considerable research is being conducted worldwide on the use of waste materials in order to avert an increasing toxic threat to the environment, or to streamline present waste disposal technique. Thus it follows that an economically viable solution is to utilize the waste materials for new products and concrete can serve as the safest home of this by product. Cement is the most cost and energy intensive component of concrete. Partial replacement of cement not only helps in conserving the raw materials used in cement manufacturing industry but also impart excellent modifications in one or more specific properties of concrete. Strength of concrete depends on a host of parameters. The experimental programme is carefully designed to evaluate the contribution of these parameters on the properties of concrete incorporating fly ash as a partial replacement of cement. In order to generate a good database, a wide range of each of these parameter is selected and the experimental work has been performed to evaluate the different properties of concrete viz. workability and strength.

A set of three binder contents (300, 375 and 450 Kg/m³), five water cementitious material ratios (0.4, 0.45, 0.5, 0.55 and 0.6),

five cement replacement percentages (0, 20, 30, 40 and 50%) have been considered. Fifty five mixes were investigated in both fresh and hardened states. Properties of fresh concrete such as slump and compacting factor were measured. A total of about 495 specimens of hardened concrete cubes have been tested for compression. After performing extensive statistical analysis of the large data base of strengths generated, simplified strength relationships for fly ash concrete have been developed.

2 EXPERIMENTAL WORK

2.1 Materials

2.1.1 Cement

In order to arrive at the desired strength ordinary portland cement of grade 43 conforming to IS: 8112 - 1989 has been used. The brand name is ultratech.

2.1.2 Fly Ash

Fly ash conforming to IS 3812-2003 & obtained from kolaghat thermal power plant has been used. The specific gravity obtained is 2.108 with a fineness of 335.52 cm²/gm.

2.1.3 Coarse Aggregates

20 mm graded coarse aggregates are rarely available in market. Hence two readily available aggregates namely 10mm single sized and 20mm single sized aggregates were mixed in different proportions by trial and error method so that the resultant sample complies to 20 mm graded aggregates as per IS: 383.

2.1.4 Fine Aggregates

For the production of strong durable concrete, good quality sand should be used. Due to incorporation of fly ash, the volume of fines in the concrete will be high. Use of Zone II sand is proved to be beneficial with regard to workability. Due to scarcity of Zone II sand in this eastern part of India, natural clean river sand conforming to Zone III as per IS 383 has been used. The specific gravity and water absorption values were obtained as 2.53 and 0.148% respectively.

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2.1.5 Water

Potable water as per is: 456-2000 used for casting and curing of concrete.

2.2 Specimen preparation, curing and testing

Compressive strengths have been measured on 150 x 150 x 150 mm cube specimens as per IS: 516. All the specimens were moist cured under water for 28 days and then air cured in the laboratory. Two different age levels have been studied – 7 and 28 days.

3 REGRESSION ANALYSIS

It is a truism to say that an improvement in the knowledge of the world around us requires an ever-increasing use of statistical methods and inferences. Statistics is the science concerned with problems involving chance variations that result from a large number of small and independent influences operating on each measured results we obtain. Strength of concrete is commonly considered its most valuable property as it gives an overall picture of the quality of concrete because it directly relates to the structure of the hydrated cement paste. Moreover, strength of concrete is almost invariably a vital element of structural design and is specified for compliance purposes. The compressive strength of concrete depends on a host of parameters and hence cannot be regarded as a fundamental or intrinsic property of the material. Therefore in any strength modeling the first step is to identify the significant parameters that affect the compressive strength of concrete. Thereafter experimentation is to be performed with those parameters to assess their effect on strength. The effects of other parameters can be eliminated by keeping them constant. It can also be checked statistically whether the parameters considered in the investigation do influence the strength of concrete or not.

3.1 Objective of the present data analysis

The fundamental idea has been to identify the most significant independent variables affecting the strength of fly ash concrete and to find out the most suitable combinations of the same. Regression analysis has been performed after selecting the primary variables and a number of strength models have been proposed exploring the different aspects of fly ash concrete with rational choice of variables and their interactions. The selection of the variables has been made by virtue of a detailed review of some of the significant relationships available in the literature.

Based on existing reporting and basics of concrete technology 13 independent, user friendly variables that can be easily measured have been selected. With these parameters 86 statistical models or mathematical equations at each age and totaling to 172 equations at 2 ages (7 and 28 days) have been investigated using the principles of regression analysis in Methods I to IV as indicated. Out of these 86 models at each

age the most efficient and statistically significant models have been identified.

3.2 Implementation of statistical tools

The value of the multiple correlation coefficient, R , measures the degree of association between the dependent variable and all of the independent variables taken together. Higher the value of R^2 better is the prediction model.

S stands for the standard estimation of error. The standard error of the estimate is a measure of the accuracy of predictions. Smaller is the standard error of estimate, the more accurate the predictions are. It is an estimate of the variability of data. Sometimes the assumed regression equation may prove to be statistically not significant. Whether this is so it cannot be properly assessed solely by the value of multiple correlation coefficient. This is checked by performing the F test, which indicates the suitability of the model in predicting the values of the dependent variable from the independent variables. If the value of F_{α} is less than F obtained the model is rendered insignificant. Similarly to test the significance of a particular independent variable on the dependent variable 't' test is performed. If a regression coefficient is found not to be statistically significant, the equation needs to be revised. The independent variable that does not significantly influence the dependent variable is deleted and new regression coefficients are computed.

3.3 Strength Modeling

This method is based on the research findings of Oluokun (1994). On the basis of the coefficients of determination (R^2), he identified the most important parameters affecting strength of fly ash concrete as the water-cementitious material ratio (w/cm), total cementitious material content (cm) and cement-fly ash ratio (c/f). He also stated that inclusion of higher order terms of these variables are not significant when considered in strength modeling. In this method, keeping w/cm constant as in all the above methods, an attempt has been made to incorporate higher order terms of these variables in strength model and to study their effects.

All possible regression analysis has been performed considering these variables

w/cm - water binder ratio

f/cm - fly ash binder ratio

cm - binder content

f/cm^2

cm^2

$2^4=16$ nos of equations have been analysed and the result interpreted.

Hence the second order regression equation involving the above variables may be written as –

$$\log(S) = a_0 + a_1 (w/cm) + a_2 (f/cm) + a_3 (cm) + a_4 (f/cm)^2 + a_5 (cm)^2$$

The 16 equations generated by combination of these above variables are provided in Table 1

TABLE 1
COMBINATION OF VARIABLES

Eq. No	Intercept	w/cm	f/cm	(cm)	(f/cm) ²	(cm) ²
1	1	1	0	0	0	0
2	1	1	0	0	0	1
3	1	1	0	0	1	0
4	1	1	0	0	1	1
5	1	1	0	1	0	0
6	1	1	0	1	0	1
7	1	1	0	1	1	0
8	1	1	0	1	1	1
9	1	1	1	0	0	0
10	1	1	1	0	0	1
11	1	1	1	0	1	0
12	1	1	1	0	1	1
13	1	1	1	1	0	0
14	1	1	1	1	0	1
15	1	1	1	1	1	0
16	1	1	1	1	1	1

** 1- indicates that the concerned parameter has been considered in regression model

** 0- indicates that the concerned parameter has not been considered in regression model

3.4 Sample Calculation

A sample calculation is provided for substantiation of the results interpreted:

For 28 day strength, for Equation 3 ,we obtain by analyzing 55 no of strength results F calculated as 208.32. From the statistical table (Ref. Table A.14 Kennedy and Neville) the value of F critical obtained at 5% level of significance is 3.175 corresponding to the degree of freedom 52:

$$D.O.F = \text{No. of samples} - (\text{No of variables} + 1) = (55 - (2 + 1)) = 52.$$

Thus $F_{\text{calculated}} > F_{\text{critical}}$ concluding that the equation is significant.

Similarly for checking the 't' statistics results, the value of t critical is obtained as 2.008(Ref. Table A.7 Kennedy and Neville) corresponding to the degree of freedom 52. The calculated value of 't' for equation 3 is greater than t critical for the intercept, w/cm as well as f/cm. Hence, the null hypothesis stands invalid and the variables of the equation is significant.

4 RESULTS AND DISCUSSIONS

4.1 Comments on R and S value:

Equations 2, 5 and 6 consisting of the variables cm and cm^2 with w/cm is rendering very poor R values as compared to other equations, hence their effect on strength modeling is not very considerable. Whereas, equations consisting of f/cm and f/cm^2 along with w/cm, is rendering much higher R values. Thus it can be concluded that the effect of f/cm and f/cm^2 is significant in strength modeling of fly ash concrete. For the 2-variable expressions, best result is obtained for Equation 3, consisting of f/cm^2 as the variable followed by Equation 9, with f/cm as the variable.

4.2 Comments on 'F' test:

For all the sixteen (16) equations for both the ages it is observed that the F values computed are greater than the critical F value at the given level of significance which indicates that at least one of the partial regression coefficients is non zero and the null hypothesis i.e., all the partial regression coefficients are zero is invalid.

4.3 Comments on 't' test

Only equations 1, 3 and 9 passes the 't' test i.e. the equations consisting of w/cm along with f/cm and f/cm^2 respectively. All other equations incorporating more than 2 variables have been found to be insignificant in the strength prediction model. The result obtained is disparate to that reported by Oluokun and higher order term of f/cm has been found significant.

Thus conclusion can be drawn on the facts that inclusion of higher no of variables does not necessarily improve the strength modeling and this fact is also evident from the earlier methods. Moreover not only f/cm but also its higher order term is significant parameter in strength modeling of fly ash concrete.

4.4 Comments on the values of regression coefficients

The a_0 values of all the equations at both the ages are positive. There is an increase in the value of the intercept at 28 days compared to 7 days..

The values of a_1 of the 16 equations at both the ages are negative indicating that at any age level and at a constant fly ash binder ratio, with increase in w/cm there is a reduction in strength, which is in accordance with Abrams w/c law.

The value of coefficient a_2 for the significant equations is negative indicating with increase in f/cm there is a reduction in strength of fly ash concrete. This is also obvious for f/cm^2 .

4.5 Final form of significant best equation

$$\log(S) = 4.9185 - 2.4486 (w/cm) - 3.0469 (f/cm)^2 \quad (1)$$

The statistical analysis has been performed for both 7 and 28 days. Here the results for R and S values are provided for 28 days from Table 2 to 6.

The t statistics data is provided in Table 7 and 8

TABLE 2

R & S VALUE FOR ALL POSSIBLE REGRESSION FOR
1 VARIABLE EQUATIONS

EQU.	VALUES OF CONSTANTS						R	S
	INTERCEPT	w/cm	f/cm	cm	(f/cm) ²	(cm) ²		
1								
7 DAYS	4.361	-2.834	0.492	0.344
28 DAYS	4.586	-2.448	0.491	0.298

TABLE 3

R & S VALUE FOR ALL POSSIBLE REGRESSION FOR
2 VARIABLE EQUATIONS

EQU.	VALUES OF CONSTANTS						R	S
	INTERCEPT	w/cm	f/cm	cm	(f/cm) ²	(cm) ²		
7 DAYS								
2	4.269	-2.734	0.000	0.493	0.347
3	4.733	-2.834	-3.445	0.930	0.147
5	4.209	-2.720	0.000	0.493	0.347
9	4.860	-2.834	-1.780	0.930	0.147
28 DAYS								
2	4.339	-2.181	7.93E-07	0.498	0.300
3	4.916	-2.449	-3.047	0.943	0.115
5	4.213	-2.168	0.001	0.498	0.300
9	5.016	-2.449	-1.533	0.925	0.131

TABLE 4

R & S VALUE FOR ALL POSSIBLE REGRESSION FOR
3 VARIABLE EQUATION

EQU.	VALUES OF CONSTANTS						R	S
	INTERCEPT	w/cm	f/cm	cm	(f/cm) ²	(cm) ²		
7 DAYS								
4	4.641	-2.734	-3.445	0.000	0.930	0.148
6	3.361	-2.720	0.005	0.000	0.495	0.350
7	4.581	-2.720	0.000	-3.445	0.930	0.148
10	4.767	-2.734	-1.780	0.000	0.930	0.148
11	4.807	-2.834	-0.915	-1.758	0.938	0.140
13	4.707	-2.720	-1.780	0.000	0.930	0.148
28 DAYS								
4	4.668	-2.181	-3.047	7.93E-07	0.946	0.113
6	3.513	-2.168	0.004	-5.09E-06	0.500	0.302
7	4.542	-2.168	0.001	-3.047	0.947	0.112
10	4.768	-2.181	-1.533	7.92E-07	0.929	0.129
11	4.946	-2.449	-0.370	-2.365	0.945	0.115
13	4.642	-2.168	-1.533	0.001	0.929	0.129

TABLE 5

R & S VALUE FOR ALL POSSIBLE REGRESSION FOR
4 VARIABLE EQUATIONS

EQU.	VALUES OF CONSTANTS						R	S
	INTERCEPT	w/cm	f/cm	cm	(f/cm) ²	(cm) ²		
7 DAYS								
8	3.733	-2.720	0.005	-3.445	0.000	0.931	0.148
12	4.715	-2.734	-0.915	-1.758	0.000	0.938	0.141
14	3.859	-2.720	-1.780	0.005	0.000	0.931	0.148
15	4.655	-2.720	-0.915	0.000	-1.758	0.938	0.141
28 DAYS								
8	3.842	-2.168	0.004	-3.047	-5.09E-06	0.948	0.113
12	4.698	-2.181	-0.370	-2.365	7.92E-07	0.948	0.112
14	3.942	-2.168	-1.533	0.004	-5.09E-06	0.930	0.129
15	4.572	-2.168	-0.370	0.001	-2.365	0.949	0.112

TABLE 6

R & S VALUE FOR ALL POSSIBLE REGRESSION FOR
5 VARIABLE EQUATIONS

EQU.	VALUES OF CONSTANTS						R	S
	INTERCEPT	w/cm	f/cm	cm	(f/cm) ²	(cm) ²		
7 DAYS								
16	3.807	-2.720	-0.915	0.005	-1.758	0.000	0.939	0.141
28 DAYS								
16	3.872	-2.168	-0.370	0.004	-2.365	-5.09E-06	0.949	0.112

TABLE 7

VALUE OF 't' AND F STATISTICS
FOR 7 DAY STRENGTH DATA

EQU.	VALUES OF 't' STATISTICS						F obs
	INTERCEPT	w/cm	f/cm	cm	(f/cm) ²	(cm) ²	
1	12.558	-4.117	16.95
2	7.808	-3.296	0.221	8.34
3	31.493	-9.638	-15.441	165.6
4	19.804	-7.732	-15.332	0.517	108.9
5	6.011	-3.277	0.251	8.35
6	1.386	-3.249	0.385	-0.366	5.52
7	15.305	-7.690	0.589	-15.344	109.17
8	3.635	-7.671	0.908	-15.306	-0.864	81.65
9	32.037	-9.647	-15.458	166.06
10	20.275	-7.739	-15.349	0.518	109.2
11	32.955	-10.133	-2.546	-2.526	124.24
12	20.976	-8.130	-2.528	-2.508	0.544	91.96
13	15.700	-7.697	-15.361	0.590	109.39
14	3.760	-7.678	-15.323	0.909	-0.864	81.82
15	16.270	-8.087	-2.530	0.619	-2.510	92.15
16	3.899	-8.073	-2.526	0.956	-2.506	-0.909	73.63

TABLE 8
VALUE OF 't' AND F STATISTICS
FOR 28 DAY STRENGTH DATA

EQU.	VALUES OF 't' STATISTICS						F obs
	INTERCEPT	w/cm	f/cm	cm	(f/cm) ²	(cm) ²	
1	15.229	-4.102	16.82
2	9.188	-3.044	0.683	8.56
3	41.737	-10.627	-17.428	208.32
4	26.162	-8.101	-17.810	1.818	146.14
5	6.967	-3.025	0.714	8.58
6	1.678	-2.999	0.405	-0.350	5.67
7	19.973	-8.069	1.906	-17.863	147.12
8	4.930	-8.060	1.089	-17.843	-0.940	110.30
9	37.017	-9.330	-14.907	154.60
10	23.193	-7.060	-15.122	1.585	106.90
11	41.376	-10.685	-1.255	-4.148	140.90
12	26.271	-8.152	-1.284	-4.242	1.830	111.40
13	17.733	-7.028	-15.156	1.660	107.40
14	4.395	-7.005	-15.107	0.947	-0.817	80.20
15	20.128	-8.121	-1.288	1.918	-4.255	112.10
16	4.999	-8.113	-1.286	1.097	-4.251	-0.946	89.70

5 CONCLUSION

It has been observed from the present study that Abrams law is very much significant (as obtained from the F and t tests results) for the present database of fly ash concrete but the efficiency of the law is not satisfactory as noted from the R value (Equation 1). Therefore age old Abrams Law has been modified here for concrete with admixtures. Thus it can be concluded that for fly ash concrete the compressive strength does not solely depend on w/cm ratio but also the other strength affecting parameters has a significant influence. The model suggests, inclusion of higher order term is effective only corresponding to f/cm while for cm it has been found to be insignificant. It has been observed from the values of the regression coefficients that corresponding to any particular age and w/cm, with increase in f/cm² there is a decrease in the strength of concrete.

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